

## UNIT-1

### Review of Number Systems & codes ①

There are four types of number systems in digital electronics

- ① Decimal number system (Base 10)
- ② Binary number system (Base 2)
- ③ Octal number system (Base 8)
- ④ Hexa-decimal number system (Base 16)

Base is also known as radix.

If the base of any number system describes the no. of distinct symbols and the highest digit/number in that number system.

Ex: In hexadecimal number system, there are 16 symbols they are

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

\* List the first 20 numbers in hexadecimal number system

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F, 10, 11, 12, 13

\* List the first 15 numbers in Base 12 system

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, 10, 11, 12

\* List the first 10 numbers in octal number system

0, 1, 2, 3, 4, 5, 6, 7, 10, 11

### Base Conversions:

To convert  $( )_{10}$  to  $( )_r$  perform the successive division of the given decimal number with required base upto the coefficient is less than the base.

Ex: Convert  $(48)_{10} = (?)_2$

(2)

2	48
	24-0
	12-0
	6-0
	3-0
	1-1

$\rightarrow (48)_{10} = (110000)_2$

\* Convert  $(71)_{10} = (?)_2$  \* Convert  $(524)_{10} = (?)_2$

2	71
	35-1
	17-1
	8-1
	4-0
	2-0
	1-0

$(71)_{10} = (1000111)_2$

2	524
	262-0
	131-0
	65-1
	32-1
	16-0
	8-0
	4-0
	2-0
	1-0

$(524)_{10} = (1000001100)_2$

\* Convert  $(461)_{10} = (?)_2$  \* Convert  $(10.125)_{10} = (?)_2$

	461
	230-1
	115-0
	57-1
	28-1
	14-0
	7-0
	3-1
	1-1

$(461)_{10} = (111001101)_2$

2	10
	5-0
	2-1
	1-0

$(10.125)_{10} = (1010.001)_2$

$\cdot 125 \times 2 = 0.250$   
 $\cdot 250 \times 2 = 0.500$   
 $\cdot 500 \times 2 = 1.000 \downarrow$

\* Convert  $(39.225)_{10} = (?)_2$

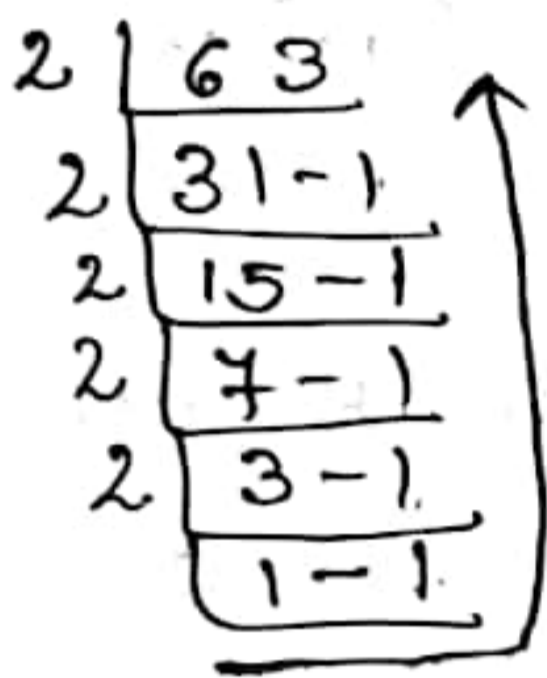
(3)



- $225 \times 2 = 0.450$
- $450 \times 2 = 0.900$
- $900 \times 2 = 1.800$
- $800 \times 2 = 1.600$

$(39.225)_{10} = (100111.0011)_2$

\* Convert  $(63.525)_{10} = (?)_2$

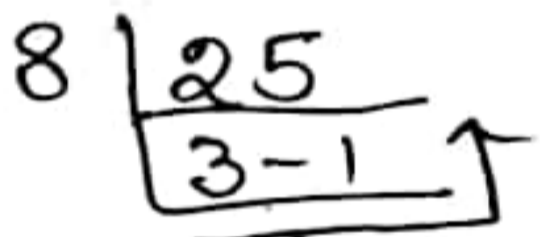


- $525 \times 2 = 1.050$
- $050 \times 2 = 0.100$
- $100 \times 2 = 0.200$
- $200 \times 2 = 0.400$
- $400 \times 2 = 0.800$

$(63.525)_{10} = (111111.1000)_2$

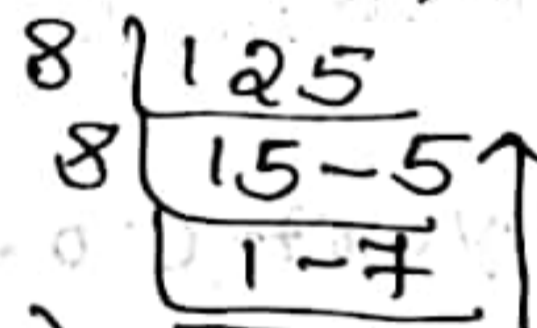
Conversion of decimal to octal

\* Convert  $(25)_{10} = (?)_8$



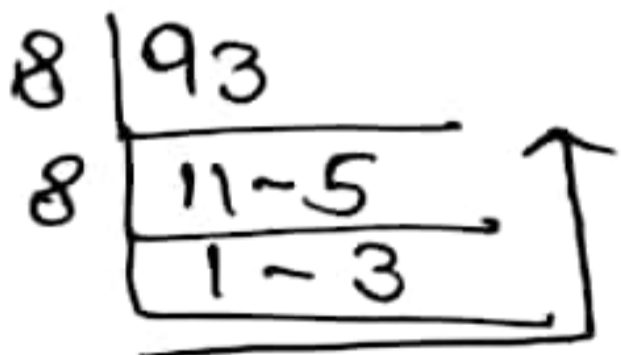
$(25)_{10} = (31)_8$

\* Convert  $(125)_{10} = (?)_8$



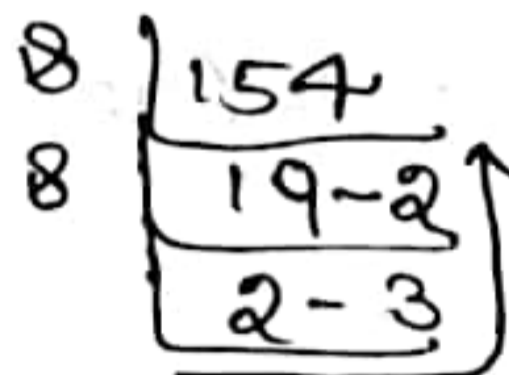
$(125)_{10} = (175)_8$

\* Convert  $(93)_{10} = (?)_8$



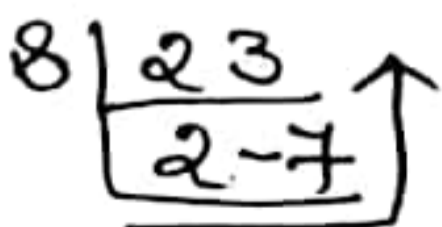
$(93)_{10} = (135)_8$

\* Convert  $(154.125)_{10} = (?)_8$



$(154.125)_{10} = (232.1)_8$

\* Convert  $(23.225)_{10} = (?)_8$



$(23.225)_{10} = (27.163)_8$

- $225 \times 8 = 1.800$
- $800 \times 8 = 6.400$
- $400 \times 8 = 3.200$
- $200 \times 8 = 1.600$



8 to 10 conversions  
Binary to decimal conversion

(5)

$$\rightarrow (110000)_2 = (?)_{10}$$

$$1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 0 \times 2^0$$

$$\Rightarrow 32 + 16$$

$$\rightarrow \boxed{(48)_{10}}$$

$$\rightarrow (1000001100)_2$$

$$1 \times 2^9 + 0 + 1 \times 2^3 + 1 \times 2^2$$

$$512 + 8 + 4$$

$$\boxed{(524)_{10}}$$

$$\rightarrow (1000111)_2$$

$$1 \times 2^6 + 0 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$$

$$64 + 4 + 2 + 1$$

$$68 + 3$$

$$\boxed{(71)_{10}}$$

$$\rightarrow (111001101)_2$$

$$1 \times 2^8 + 1 \times 2^7 + 1 \times 2^6 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^0$$

$$256 + 128 + 64 + 8 + 4 + 2 + 1$$

$$\boxed{(461)_{10}}$$

$$\rightarrow (1010.001)_2 = (?)_{10}$$

$$1 \times 2^3 + 1 \times 2^1 + 0 \times 2^{-1} + 1 \times 2^{-3}$$

$$8 + 2 + \frac{1}{2^3} \Rightarrow 10 + 0.125$$

$$\boxed{(10.125)_{10}}$$

$$\rightarrow (100111.0011)_2 = (?)_{10}$$

$$1 \times 2^5 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-3} + 1 \times 2^{-4}$$

$$\Rightarrow 32 + 4 + 2 + 1 + \frac{1}{2^3} + \frac{1}{2^4}$$

$$\Rightarrow 39 + 0.125 + 0.0625 \Rightarrow (39.1875)_{10} \approx \boxed{(39.225)_{10}}$$

$$\rightarrow (111111.1000)_2 = (?)_{10}$$

$$1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1}$$

$$32 + 16 + 8 + 4 + 2 + 1 + 0.5 \Rightarrow \boxed{(63.5)_{10}}$$

# Octal to decimal conversion

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\*  $(31)_8 = (?)_{10}$

$$8^1 \times 3 + 1 \times 8^0$$

$$24 + 1$$

$$\boxed{(25)_{10}}$$

\*  $(175)_8 = (?)_{10}$

$$1 \times 8^2 + 7 \times 8^1 + 5 \times 8^0$$

$$64 + 56 + 5$$

$$\boxed{(125)_{10}}$$

\*  $(135)_8 = (?)_{10}$

$$1 \times 8^2 + 3 \times 8^1 + 5 \times 8^0$$

$$64 + 24 + 5$$

$$\boxed{(93)_{10}}$$

\*  $(27.1631)_8 = (?)_{10}$

$$2 \times 8^1 + 7 \times 8^0 + 1 \times 8^{-1} + 6 \times 8^{-2} + 3 \times 8^{-3} + 1 \times 8^{-4}$$

$$16 + 7 + 0.125 + 0.09375 +$$

$$0.00585 + 0.00024$$

$$\boxed{(23.2248)_{10}}$$

\*  $(232.1)_8 = (?)_{10}$

$$2 \times 8^2 + 3 \times 8^1 + 2 \times 8^0 + 1 \times 8^{-1}$$

$$128 + 24 + 2 \times \frac{1}{8}$$

$$154 + 0.125$$

$$\boxed{154.125}$$

## hexadecimal to decimal

→  $(1F)_{16} = (?)_{10}$

$$1 \times 16^1 + 15 \times 16^0$$

$$16 + 15 \Rightarrow \boxed{(31)_{10}}$$

→  $(3E)_{16} = (?)_{10}$

$$3 \times 16^1 + E \times 16^0$$

$$48 + 14 \times 1$$

$$\boxed{(62)_{10}}$$

→  $(4A)_{16} = (?)_{10}$

$$4 \times 16^1 + A \times 16^0$$

$$64 + 10 \times 1$$

$$\boxed{(74)_{10}}$$

→  $(31)_{16} = (?)_{10}$

$$16^1 \times 3 + 16^0 \times 1$$

$$48 + 1 = \boxed{(49)_{10}}$$

→  $(46.4)_{16} = (?)_{10}$

$$4 \times 16^1 + 6 \times 16^0 + 4 \times \frac{1}{16}$$

$$64 + 6 + 0.250$$

$$\Rightarrow \boxed{(70.250)_{10}}$$

\*  $(5C.2666)_{16} = (?)_{10}$

$$5 \times 16^1 + 12 \times 16^0 + 2 \times 16^{-1} + 6 \times 16^{-2} + 6 \times 16^{-3} + 6 \times 16^{-4}$$

$$80 + 12 + 0.125 + 0.0234 + 0.0014 + 0.00009$$

$$\Rightarrow (92.1498)_{10} \approx \boxed{(92.150)_{10}}$$

# Base 4 to Base 10

(7)

\*  $(22.02)_4$

$2 \times 4^1 + 2 \times 4^0 + 0 + 2 \times 4^{-2}$

$8 + 2 + 0 + 2 \times \frac{1}{16}$

$10 + 0.125$

$(10.125)_{10}$

## γ-γ Conversions:

In binary to octal conversion segment the given binary number as group and each group should contain 3 bit. Replace each group with its octal equivalent.

3-bit binary code

2	1	0	
2	2	2	
4	2	1	
0	0	0	- 0
0	0	1	- 1
0	1	0	- 2
0	1	1	- 3
1	0	0	- 4
1	0	1	- 5
1	1	0	- 6
1	1	1	- 7

$2^3 = 8$

\* Convert  $(1011010)_2$  to  $(?)_8$

$001 | 011 | 010$   
 1 | 3 | 2

$(132)_8$

\*  $(110101101101)_2$  to  $(?)_8$

$001 | 110 | 101 | 101 | 101$   
 1 | 6 | 5 | 5 | 5

$(16555)_8$

\*  $(11101111110110.0110111)_2 = (?)_8$  (8)

111|011|111|110|110.011|011|100  
 7 3 7 6 6 . 3 3 4

$(73766.334)_8$

Binary to hexadecimal:

4 bit binary code

$2^3$	$2^2$	$2^1$	$2^0$	
0	0	0	0	- 0
0	0	0	1	- 1
0	0	1	0	- 2
0	0	1	1	- 3
0	1	0	0	- 4
0	1	0	1	- 5
0	1	1	0	- 6
0	1	1	1	- 7
1	0	0	0	- 8
1	0	0	1	- 9
1	0	1	0	- 10 - A
1	0	1	1	- 11 - B
1	1	0	0	- 12 - C
1	1	0	1	- 13 - D
1	1	1	0	- 14 - E
1	1	1	1	- 15 - F

$2^4 = 16$



\*  $(11001010)_2 = (?)_{16}$

\* (10

9

$$\begin{array}{c|c} 1100 & 1010 \\ \hline C & A \end{array}$$

$(CA)_{16}$

\*  $(10101110101011)_2 = (?)_{16}$

$$\begin{array}{c|c|c|c} 0010 & 1011 & 1010 & 1011 \\ \hline 2 & B & A & B \end{array}$$

$(2BAB)_{16}$

\*  $(101101010.101010111)_2 = (?)_{16}$

$$\begin{array}{c|c|c|c|c|c} 0001 & 0110 & 1010 & . & 1010 & 1011 & 1000 \\ \hline 1 & 6 & A & & A & B & 8 \end{array}$$

$(16A.AB8)_{16}$

0001000000010001

Octal to Binary conversion

\* In octal to Binary conversion each digit of octal number should be replaced with its 3-bit binary equivalent

\*  $(132)_8$

$$\begin{array}{c|c|c} 1 & 3 & 2 \\ \hline 001 & 011 & 010 \end{array}$$

$(001011010)_2$

\*  $(16555)_8$

$$\begin{array}{c|c|c|c|c} 1 & 6 & 5 & 5 & 5 \\ \hline 001 & 110 & 101 & 101 & 101 \end{array}$$

\*  $(73766.334)_8$

$$\begin{array}{c|c|c|c|c|c|c|c} 7 & 3 & 7 & 6 & 6 & . & 3 & 3 & 4 \\ \hline 111 & 011 & 111 & 110 & 110 & . & 011 & 011 & 100 \end{array}$$

$(11101111110110.011011)_2$

## Hexadecimal to binary:

$$*(1011)_{16} = (?)_2$$

$$\begin{array}{c|c|c|c} 1 & 0 & 1 & 1 \\ \hline 0001 & 0000 & 0001 & 0001 \end{array}$$

$$(0001000000010001)_2$$

$$*(CA)_{16} = (?)_2$$

$$\begin{array}{c|c} C & A \\ \hline 1100 & 1010 \end{array}$$

$$(11001010)_2$$

## Octal to hexadecimal conversion:

\* to convert Octal to hexadecimal convert the octal number to binary and then to hexadecimal.

$$\rightarrow \text{convert } (45)_8 = (?)_{16}$$

$$(a) (145)_8 = (?)_2$$

$$\begin{array}{c|c|c} 1 & 4 & 5 \\ \hline 001 & 100 & 101 \end{array}$$
$$(001100101)_2$$

$$(b) \begin{array}{c|c|c} 0000 & 0110 & 0101 \\ \hline 0 & 6 & 5 \end{array}$$

$$(065)_{16} = (65)_{16}$$

$$\rightarrow (1372)_8 = (?)_{16}$$

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(a)  $(1372)_8 = (?)_2$

$$001 \mid 011 \mid 111 \mid 010$$

$$(001011111010)_2$$

(b)  $(001011111010)_2 = (?)_{16}$

$$0010 \mid 1111 \mid 1010$$

2 F A

$$(2FA)_{16}$$

Hexadecimal to octal

$$\rightarrow (DAC)_{16} = (?)_8$$

(a)  $(DAC)_{16} = (?)_2$

$$D \mid A \mid C$$

$$1101 \mid 1010 \mid 1100$$

$$(110110101100)_2$$

(b)  $(110 \mid 110 \mid 101 \mid 100)_2 = (?)_8$

$$(6654)_8$$

\*Note

10-8 → successive division

8-10 → powers multiplication

8-8 → if powers relation exists then grouping and use bit code else

use intermediate (decimal / binary) to convert

$$\rightarrow (1A8)_{16} = (?)_8$$

(a)  $(1A8)_{16} = (?)_2$

$$1 \mid A \mid 8$$

$$0001 \mid 1010 \mid 1000$$

$$(000110101000)_2$$

(b)  $000 \mid 110 \mid 101 \mid 000$

0 6 5 0

$$(650)_8$$

# Binary addition:

→ 8 → 1 0 0 0

4 → 0 1 1 1

15 → 1 1 1 1

→ 7 0 1 1 1

7 0 1 1 1

14 1 1 1 0

→ 15 1 1 1 1

15 1 1 1 1

1 1 1 1 0

→ 5 0 1 0 1

5 0 1 0 1

10 1 0 1 0

→ 10 1 0 1 0

1 1 1 1

0 1 1 1

1 0 0 0 0 0

Here 2 → 10

3 → 0 0 1 1

4 → 0 1 0 0

5 → 0 1 0 1

6 → 0 1 1 0

7 → 0 1 1 1

## Binary subtraction:

8 1 0 0

- 4 0 1 0

4 0 1 0

0 1 0 1

0 1 0 1

0 1 0 1

2 1 0

7 0 1 1 1

1 8 0 0 0 0

6 1 1 0 8 1 0 0 0

2 0 1 0 7 0 1 1 1

4 1 0 0 1 0 0 0 1

1 0 0 1 0 0 0 1

$$\begin{array}{r}
 \begin{array}{cccccc}
 & 0 & 1 & 2 & 2 & 2 \\
 * & 16 & 1 & 0 & 0 & 0 \\
 & 14 & 0 & 1 & 1 & 1 \\
 \hline
 & 2 & 0 & 0 & 0 & 1 \\
 \hline
 & & & & & 0
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{cccccc}
 * & 18 & 1 & 0 & 0 & 1 \\
 & 13 & 0 & 1 & 1 & 0 \\
 \hline
 & 5 & 0 & 0 & 1 & 0 \\
 \hline
 & & & & & 1
 \end{array}
 \end{array}$$

Compliments:

For any base "r" system there exists two compliments  
1. r's compliment  
2. (r-1)'s compliment

\* To find r's Compliment the following formula is used i.e.,  $r^n - N$

Where r - Base of number system  
n - no. of integer digits in the given number  
N - The number for which r's compliment to be calculated.

\* To find (r-1)'s compliment the following formula is used i.e.,  $(r^n - 1) - N$

Compliments in binary number system.

1's compliment

→ Find 1's compliment of 101

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$$\begin{array}{l}
 N = 101 \\
 n = 3, r = 2
 \end{array}$$

$$\text{Formula: } (r^n - 1) - N$$

$$(2^3 - 1) - N$$

$$(8 - 1) - 101$$

$$7(111) - 101$$

$$111 - 101$$

$$\boxed{010}$$

$$\begin{array}{r}
 111 \\
 101 \\
 \hline
 010
 \end{array}$$

1's compliment of 101 is 010

→ Find 1's complement of 1010

Simply replace 1's with 0 & 0 with 1.

1's complement of 1010 is 0101

(14)

→ 100011

1's complement is 011100

→ 2's complement

2's complement can be calculated by adding 1 to 1's complement of number.

\* Find the 2's complement of following number

→ 1101

1's complement = 0010

Add 1 = 1

2's complement 0011

\* To find 2's complement directly there are two cases.

(i) if units place have zero then from right to left do not change the number until 1 is reached then next numbers are interchanged by 0's as 1 & 1's as 0

→ 1000

1's complement = 0111

Add 1 = 1

2's complement 1000

(ii) if units place have 1 then not exchange 1 kept it as 1 & then next numbers are interchanged by 0 as 1 & 1 as 0

→ 10000

1's complement = 01111

Add 1 = 1

10000

→ 10110

1's complement = 01001

01010

By directly 01010

(i) 1010  
2's comp Ans 01010

(ii) 111101

2's comp 000011

→ 10000

1's complement = 01111

10000



\*  $10 \rightarrow 1010$   
 (-)  $7 \rightarrow 0111$   
 ---  
 3

$10 \rightarrow 1010$   
 1's comp of 7 (+)  $0000$   
 ---  
 10010  
 (+)  $1$   
 ---  
 0011

$7 \rightarrow 0111$   
 10  $0101$   
 ---  
 -3  $1100$   
 ---  
 -0011

\*  $15 \rightarrow 1111$   
 (-)  $11 \rightarrow 1011$   
 ---  
 4  $0100$

$15 \rightarrow 1111$   
 1's comp of 11 (+)  $0100$   
 ---  
 10011  
 (+)  $1$   
 ---  
 00100

$11 \rightarrow 1011$   
 1's of 15  $0000$   
 ---  
 -4  $1011$   
 ---  
 -0100

\*  $101101$   
 $010110$   
 ---  
 ---

case (i)

$101101$   
 (+)  $101001$   
 ---  
 1010110  
 (+)  $1$   
 ---  
 010111

$010110$   
 (+)  $010010$   
 ---  
 101000  
 ---  
 (-)  $010111$



# 2's Complement Subtraction:

(17)

If "A-B" is the required operation, take the two's complement of "B" and add it to "A". After addition there are two cases.

## Case (i):

Carry generated. After adding "A" & 2's complement of "B" if carry is generated <sup>neglect</sup> discard the carry.

## Case (ii):

Carry is not generated. After adding "A" & 2's complement of "B" if carry is not generated take the 2's complement of result again and put a (-) (minus) sign before it.

## Ex:

$$*8 \rightarrow 1000 \rightarrow 1000$$

$$(-)5 \rightarrow 0101 \rightarrow (+)1011$$

$$\underline{3}$$

$$\begin{array}{r} 10011 \\ \underline{0011} \\ \hline \end{array}$$

discard the carry

Ans. 3

$$5 \rightarrow 0101 \rightarrow$$

$$(-)8 \rightarrow 1000$$

$$\underline{-3}$$

$$0101$$

$$(+)\underline{1000}$$

$$\underline{1101}$$

$$\text{As } 2's \text{ } (-0011) \rightarrow -3$$

$$*9 \rightarrow 1001 \rightarrow$$

$$(-)4 \rightarrow 0100$$

$$\underline{5}$$

$$1001$$

$$(+)\underline{1100}$$

$$\underline{10101}$$

discard

$$0101 \rightarrow 5$$

$$4 \rightarrow 0100 \rightarrow$$

$$(-)9 \rightarrow 1001$$

$$\underline{-5}$$

$$0100$$

$$(+)\underline{0111}$$

$$\underline{10111}$$

$$2's \text{ } (-0101) \rightarrow -5$$

\*  $^{23} 1011 \rightarrow 1011$  (8)  
 $^{10} (-) 01010$   
 $(+) 10110$   
 discard  $01101$

$^{10} 01010 \rightarrow 01010$   
 $^{23} (-) 10111$   
 $(+) 01001$   
 $10011$   
 $(-01101)$

\*  $^{32} 001101 \rightarrow 001101$   
 $^{13} 111101$   
 $(+) 000011$   
 $010000$   
 $-110000$

$111101 \rightarrow 111101$   
 $001101$   
 $(+) 110011$   
 $110000$   
 discard  $110000$

9's Complement subtraction:

If 'A-B' is the required operation Take the 9's complement of B & add it to A. After adding there are two cases

Case 1: Carry generated.

If Carry generated after the addition of A and 9's complement of B. End around the carry i.e., add carry to LSB

Case 2: Carry not generated

If Carry not generated after the addition take the 9's complement of result again and put a '-' sign before the result.

\*  $8 \rightarrow 8$   
 $-5 \xrightarrow{9's\ comp} 4$   
 $\begin{array}{r} 8 \\ -5 \\ \hline 3 \end{array}$   
 end a round carry

$\begin{array}{r} 9 \\ -5 \\ \hline 4 \end{array}$

$5 \rightarrow 5$   
 $-8 \xrightarrow{9's\ comp} 1$   
 $\begin{array}{r} 5 \\ -8 \\ \hline 6 \end{array}$   
 9's of 6  $\boxed{-3}$

$\begin{array}{r} 9 \\ -8 \\ \hline 1 \end{array}$   
 $\begin{array}{r} 9 \\ -6 \\ \hline 3 \end{array}$

\*  $9 \rightarrow 9$   
 $-4 \xrightarrow{9's\ comp} 5$   
 $\begin{array}{r} 9 \\ -4 \\ \hline 5 \end{array}$

$\begin{array}{r} 9 \\ -4 \\ \hline 5 \end{array}$

$4 \rightarrow 4$   
 $-9 \xrightarrow{9's\ comp} 0$   
 $\begin{array}{r} 4 \\ -9 \\ \hline 4 \end{array}$   
 9's of 4  $\boxed{-5}$

$\begin{array}{r} 9 \\ -4 \\ \hline 5 \end{array}$

\*  $25 \rightarrow 25$   
 $-19 \xrightarrow{9's\ comp} 80$   
 $\begin{array}{r} 25 \\ -19 \\ \hline 6 \end{array}$

$\begin{array}{r} 99 \\ -19 \\ \hline 80 \end{array}$

$19 \rightarrow 19$   
 $-25 \xrightarrow{9's\ comp} 74$   
 $\begin{array}{r} 19 \\ -25 \\ \hline 74 \end{array}$   
 $\begin{array}{r} 99 \\ -25 \\ \hline 74 \end{array}$   
 $\begin{array}{r} 99 \\ -74 \\ \hline 25 \end{array}$   
 $\begin{array}{r} 99 \\ -93 \\ \hline 6 \end{array}$

$\begin{array}{r} 99 \\ -25 \\ \hline 74 \end{array}$   
 $\begin{array}{r} 99 \\ -93 \\ \hline 6 \end{array}$

\*  $84 \rightarrow 84$   
 $-27 \xrightarrow{9's\ comp} 72$   
 $\begin{array}{r} 84 \\ -27 \\ \hline 57 \end{array}$

$\begin{array}{r} 99 \\ -27 \\ \hline 72 \end{array}$

$27 \rightarrow 27$   
 $(-84) \rightarrow 15$   
 $\begin{array}{r} 27 \\ -84 \\ \hline 15 \end{array}$   
 $\begin{array}{r} 99 \\ -84 \\ \hline 15 \end{array}$   
 9's of 42  $\boxed{-57}$

$\begin{array}{r} 99 \\ -84 \\ \hline 15 \end{array}$   
 $\begin{array}{r} 99 \\ -42 \\ \hline 57 \end{array}$

1.  
 \*  $459 \rightarrow 459$   
 $(-362) \xrightarrow{9's\ comp} 637$   
 $\begin{array}{r} 459 \\ -362 \\ \hline 097 \end{array}$

$\begin{array}{r} 999 \\ -362 \\ \hline 637 \end{array}$

$362 \rightarrow 362$   
 $(-459) \rightarrow 540$   
 $\begin{array}{r} 362 \\ -459 \\ \hline 902 \end{array}$   
 9's of 902  $\boxed{-97}$   
 $\begin{array}{r} 999 \\ -459 \\ \hline 540 \end{array}$   
 $\begin{array}{r} 999 \\ -902 \\ \hline 97 \end{array}$

# 10's Complement Subtraction

(20)

If "A-B" is the required operation, takes 10's Complement of B and add it to A. After addition there are two cases:

Case (i) - Carry generated

If carry generated after adding A & 10's Complement of B, if carry generated then discard the carry

Case (ii) - Carry not generated.

After adding A & 10's Complement of B if carry not generated take 10's Complement of result again and put a (-) minus sign before it.

<p>* <math display="block">\begin{array}{r} 9 \rightarrow 9 \\ (-) 3 \xrightarrow{10's\ comp} 7 \\ \hline (+) 6 \\ \hline \text{discard } 6 \\ \hline \end{array}</math></p> <p><math display="block">\begin{array}{r} 10's\ comp \\ 9 \\ (-) 3 \\ \hline 6 \\ + 1 \\ \hline \boxed{7} \end{array}</math></p>	<p><math display="block">\begin{array}{r} 3 \rightarrow 3 \\ - 9 \xrightarrow{10's\ comp} 1 \\ \hline 10's\ comp\ 4 \\ \hline \boxed{-6} \end{array}</math></p> <p><math display="block">\begin{array}{r} 10's\ comp \\ 9 \\ - 9 \\ \hline 0 \\ + 1 \\ \hline 1 \end{array}</math></p>
<p>* <math display="block">\begin{array}{r} 8 \rightarrow 8 \\ (-) 4 \xrightarrow{10's\ comp} 6 \\ \hline (+) 4 \\ \hline \text{discard } 4 \\ \hline \end{array}</math></p> <p><math display="block">\begin{array}{r} 10's\ comp \\ 9 \\ (-) 4 \\ \hline 5 \\ + 1 \\ \hline 6 \end{array}</math></p>	<p><math display="block">\begin{array}{r} 4 \rightarrow 4 \\ - 8 \xrightarrow{10's\ comp} 2 \\ \hline 10's\ comp\ 6 \\ \hline \boxed{-4} \end{array}</math></p> <p><math display="block">\begin{array}{r} 10's\ comp \\ 9 \\ - 8 \\ \hline 1 \\ + 1 \\ \hline 2 \end{array}</math></p>
<p>* <math display="block">\begin{array}{r} 56 \rightarrow 56 \\ - 43 \xrightarrow{10's\ comp} 57 \\ \hline (+) 1 \\ \hline \text{discard } 1 \\ \hline \end{array}</math></p> <p><math display="block">\begin{array}{r} 10's\ comp \\ 99 \\ - 43 \\ \hline 56 \\ + 1 \\ \hline 57 \end{array}</math></p>	<p><math display="block">\begin{array}{r} 43 \rightarrow 43 \\ - (56) \xrightarrow{10's\ comp} 44 \\ \hline (+) 1 \\ \hline 87 \\ 10's\ comp\ 87 \rightarrow \boxed{-13} \end{array}</math></p> <p><math display="block">\begin{array}{r} 10's\ comp \\ 99 \\ 56 \\ \hline 43 \\ + 1 \\ \hline 44 \end{array}</math></p>
	<p><math display="block">\begin{array}{r} 99 \\ - 87 \\ \hline 12 \\ + 1 \\ \hline 13 \end{array}</math></p>



2 is the is compliment of 8<sup>x</sup> and so on.

Ex: 2421

(22)

→ Write the 2421 Code

	2	4	2	1
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	1	1	1	1
6	1	1	0	0
7	1	1	0	1
8	1	1	1	0
9	1	1	1	1

Alphanumeric codes:

In alphanumeric codes, each alphabet (both uppercase & lowercase), numbers and symbols are having specific binary representation.

Ex: ASCII code

American Standard Code for Information Interchange

# Error Correcting & Error detecting codes <sup>(23)</sup>

By using some of the binary codes errors can be detected but not corrected they are called error detecting codes.

By using some of the binary codes errors can be detected & corrected they are called error correcting code.

Ex: Hamming code.

## BCD Code (Binary Coded decimal)

In BCD code all the decimal digits from 0 to 9 are coded with binary numbers as shown below

	8	4	2	1
0-0	0	0	0	0
1-0	0	0	0	1
2-0	0	1	0	0
3-0	0	1	1	0
4-0	1	0	0	0
5-0	1	0	1	0
6-0	1	1	0	0
7-0	1	1	1	0
8-1	0	0	0	1
9-1	0	0	1	1

# BCD addition

(24)

In BCD addition, after adding two BCD numbers, the result should be valid BCD number. If it is invalid add correction factor 6 (0110) to the invalid BCD number.

$$\begin{array}{r}
 * 5 \rightarrow 0101 \\
 3 \rightarrow 0011 \\
 \hline
 8 \rightarrow 1000
 \end{array}$$

$$\begin{array}{r}
 * 5 \rightarrow 0101 \\
 \underline{5} \rightarrow 0101 \\
 10 \rightarrow 1010 \rightarrow \text{Invalid BCD} \\
 \hline
 +6 \rightarrow 0110 \\
 \hline
 0001 \quad 0000 \\
 \hline
 1 \quad 0
 \end{array}$$

$$\begin{array}{r}
 * 8 \rightarrow 1000 \\
 +6 \rightarrow 0110 \\
 \hline
 14 \rightarrow 1110 \rightarrow \text{Invalid BCD} \\
 \hline
 +6 \rightarrow 0110 \\
 \hline
 0001 \quad 0100 \\
 \hline
 1 \quad 4
 \end{array}$$

$$\begin{array}{r}
 * 15 \rightarrow 00010101 \\
 \underline{12} \rightarrow 00010010 \\
 27 \rightarrow 00100111 \\
 \hline
 \quad \quad 2 \quad 7
 \end{array}$$

$$\begin{array}{r}
 * 39 \rightarrow 00111001 \\
 \underline{23} \rightarrow 00100011 \\
 62 \rightarrow 01011100 \rightarrow \text{Invalid BCD} \\
 \hline
 +6 \rightarrow 0110 \\
 \hline
 0110 \quad 0010 \\
 \hline
 6 \quad 2
 \end{array}$$



\* 28 → 0010 1000  
 + 26 → (+) 0010 0110

0101 0100 +6  
 5 4

0100 1110 → Invalid BCD  
 0101 0100  
 5 4

\* 45 → 0100 0101  
 + 38 → 0011 1000

0111 1101 → Invalid BCD

1000 0011 +6  
 8 3

1111 0110  
 1000 0011  
 8 3

\* 157 → 0001 0101 0111  
 + 138 0001 0011 1000

295 0010 1000 1111

0010 1001 0101 +6  
 2 9 5

0010 1001 0101  
 2 9 5

\* 867 → 1000 0110 0111  
 + 574 0101 0111 0100

1441 1101 1101 1011

0001 0100 0100 0001 +6  
 1441 1101 1110 0001

+6 110110  
 1110 0100 0001

+6 0110  
 0001 0100 0100 0001  
 1 4 4 1

# BCD Subtraction:

(26)

If "(A-B)" is required operation, take the 9's complement of "B" and add it to A. After adding there are two cases.

## Case 1: Invalid BCD

If the sum is invalid BCD number validate it by adding 6 (0110). After addition of 6, the carry has to be end arounded.

## Case 2: Valid BCD

After addition, if the result is valid BCD, take the 9's complement of result once again & put (-) minus sign before it.

Ex:

①

$$\begin{array}{r} 8 \rightarrow 1000 \\ (-) 4 \xrightarrow{9's\ complement} 0101 \\ \hline 4 \\ 0100 \end{array} \quad \begin{array}{r} (+) \\ 1101 \rightarrow \text{Invalid BCD} \\ + 6 \ 0110 \\ \hline 1001 \\ \hline 111 \\ + \\ \hline 0100 \end{array}$$

$$\begin{array}{r} 4 \rightarrow 0100 \\ - 8 \xrightarrow{9's\ complement} 0001 \\ \hline 4 \\ - 0100 \\ \hline 0101 \rightarrow \text{Valid BCD} \\ \swarrow \\ 9's\ complement \rightarrow \boxed{0100} \end{array}$$

②

$$\begin{array}{r} 9 \rightarrow 1001 \\ - 3 \xrightarrow{9's\ complement} 0110 \\ \hline 6 \\ 0110 \end{array} \quad \begin{array}{r} 9 \\ - 3 \\ \hline 6 \\ 1111 \rightarrow \text{Invalid BCD} \\ + 6 \ 0110 \\ \hline 1010 \\ \hline 111 \\ + \\ \hline 0110 \end{array}$$

③

$$\begin{array}{r} 3 \rightarrow 0011 \\ - 9 \xrightarrow{9's\ complement} 0000 \\ \hline 6 \\ - 6 \\ \hline 0011 \\ \swarrow \\ 9's\ complement \text{ is } \boxed{-0110} \end{array}$$



$$\begin{array}{r}
 143 \rightarrow 0001 \quad 0100 \quad 0011 \\
 -257 \rightarrow 0111 \quad 0100 \quad 0010 \\
 \hline
 -114 \rightarrow \begin{array}{r} + \\ 111 \\ \hline 1000 \quad 1000 \quad 0101 \end{array}
 \end{array}$$

$$\begin{array}{r}
 999 \\
 257 \\
 \hline
 742
 \end{array}$$

$$\begin{array}{r}
 999 \\
 885 \\
 \hline
 114
 \end{array}$$

9's of 885 is -0001 0001 0100

## Gray Code:

Gray Code is a non weighted code, derived from binary code. In gray code there is only one bit change from one number to the immediate next number.

Q. Generate 4-bit gray code from single bit gray code.

Ans

$$\begin{array}{r}
 Q-0 \quad *Q \quad Q \\
 1-1 \quad *Q \quad 1 \\
 \hline
 *Q \quad 1 \\
 *Q \quad 0
 \end{array}$$

0-0	0-00	0-00 0	0-00 0 0
1-1	1-01	1-00 1	1-00 0 1
	2-11	2-01 1	2-00 1 1
	3-10	3-01 0	3-00 1 0
		4-11 0	4-01 1 0
		5-11 1	5-01 1 1
		6-10 1	6-01 0 1
		7-10 0	7-01 0 0
			8-11 0 0
			9-11 0 1
			10-11 1 1
			11-11 1 0
			12-10 1 0
			13-10 1 1
			14-10 0 1
			15-10 0 0

# Binary to Gray code

To convert given 4-bit binary to its equivalent 4-bit gray code, the following formula is used.

Let the given 4-bit binary number is  $(B_3 B_2 B_1 B_0)$  and its equivalent 4-bit gray code is  $(G_3 G_2 G_1 G_0)$  then

$$G_3 = B_3$$

$$G_2 = B_3 \oplus B_2$$

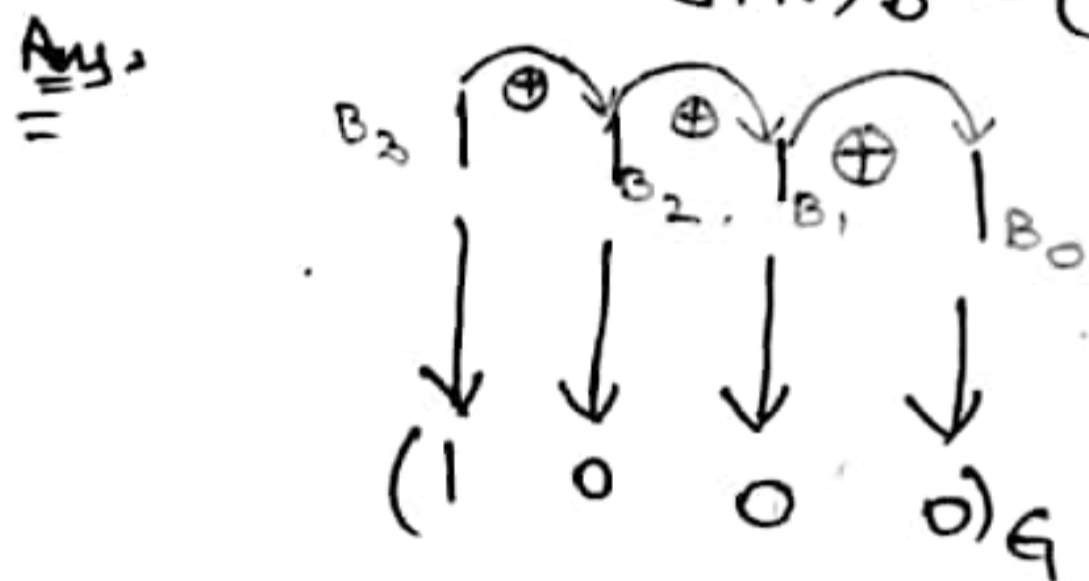
$$G_1 = B_2 \oplus B_1$$

$$G_0 = B_1 \oplus B_0$$

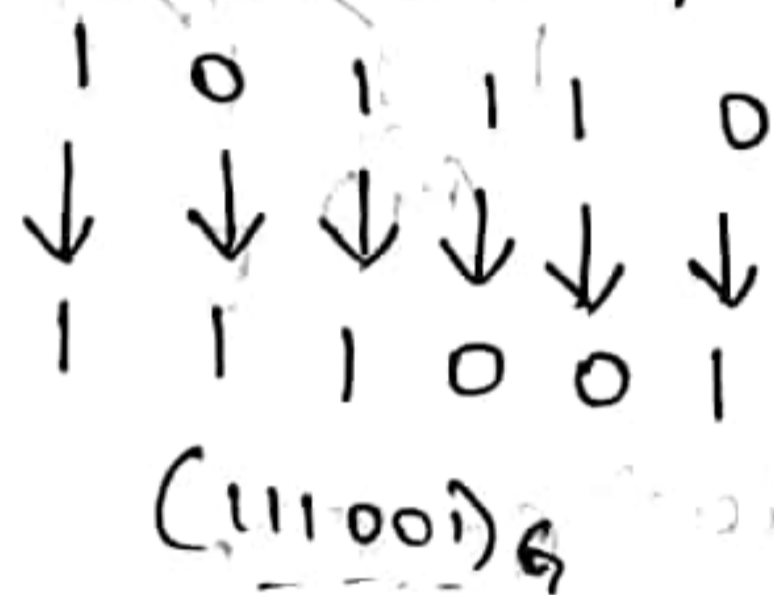
EX-OR truth table

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

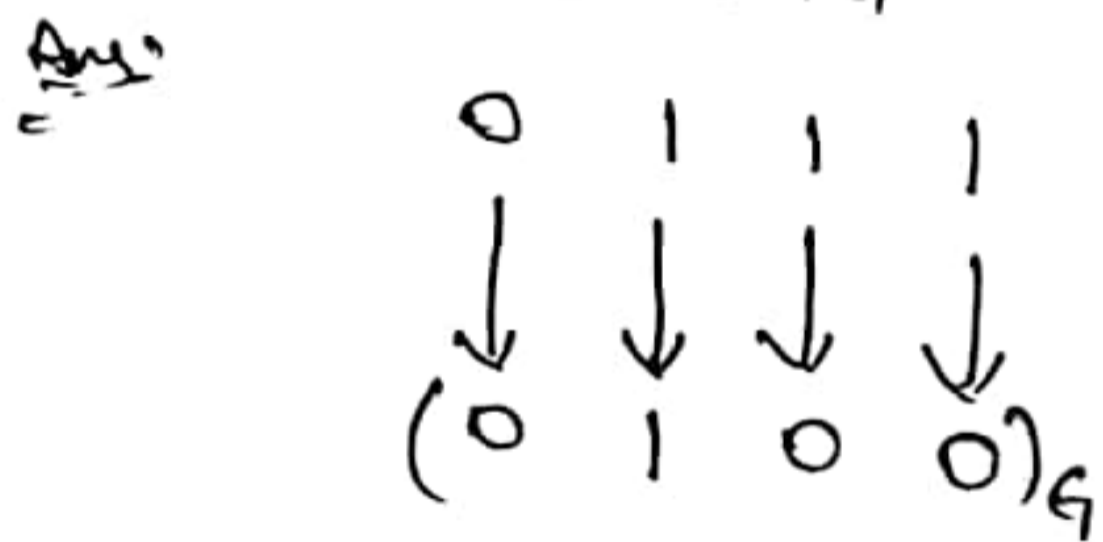
→ Convert  $(1111)_B = (?)_G$



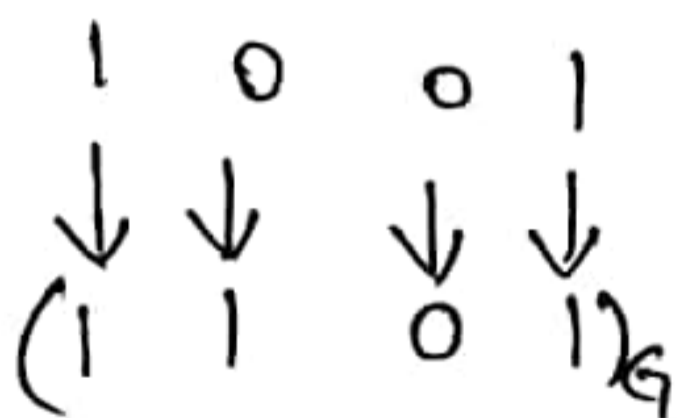
→  $(101110)_B = (?)_G$



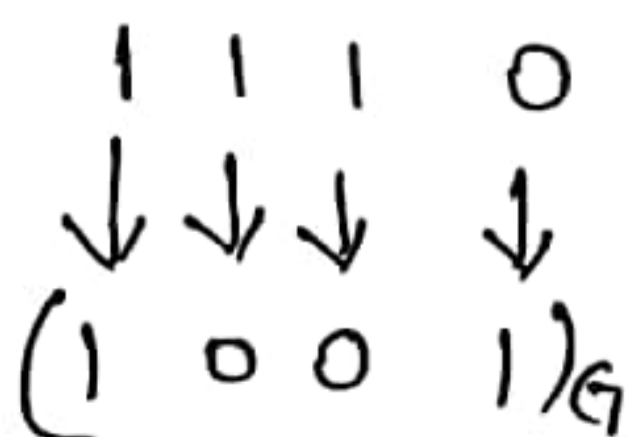
→  $(0111)_B = (?)_G$



→  $(1001)_B = (?)_G$



→  $(1110)_B = (?)_G$



# Gray code to Binary code Conversion:

30

Let the given 4-bit gray code is  $(G_3 G_2 G_1 G_0)$  and its equivalent binary is  $(B_3 B_2 B_1 B_0)$  then

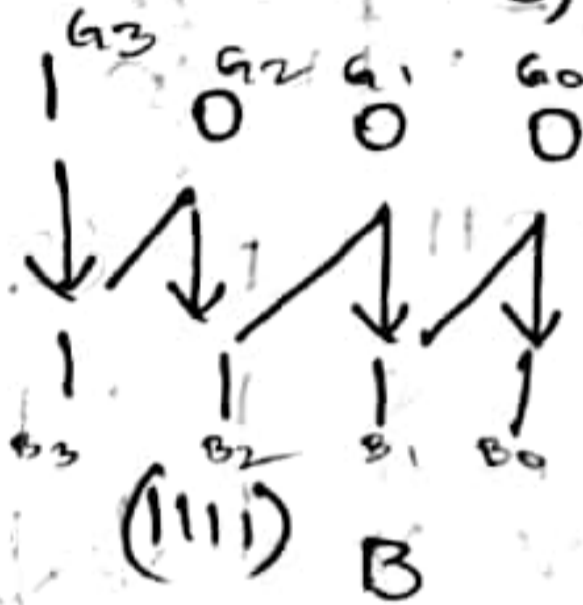
$$B_3 = G_3$$

$$B_2 = G_2 \oplus B_3$$

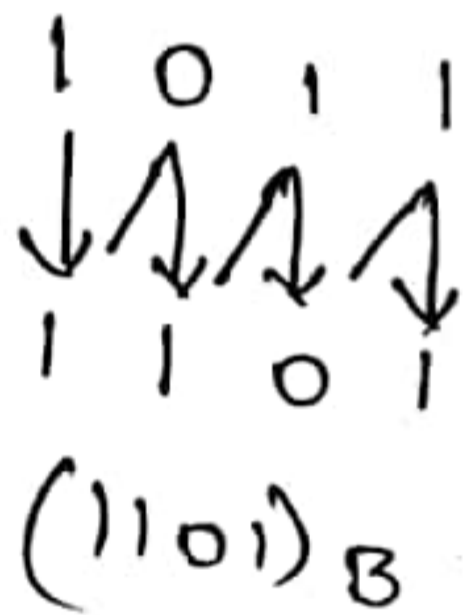
$$B_1 = B_2 \oplus G_1$$

$$B_0 = B_1 \oplus G_0$$

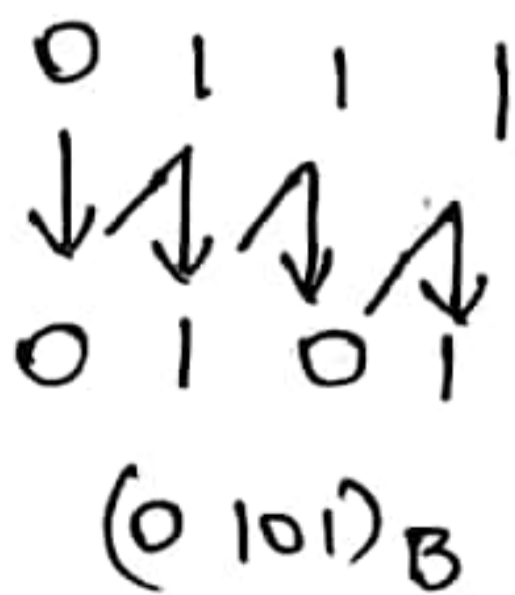
→ Convert  $(1000)_G = (?)_B$



→  $(1011)_G = (?)_B$



→  $(0111)_G = (?)_B$



### Excess-3 code

Excess-3 code is a non weighted code delivered from BCD code for every BCD number 3 is added to get excess 3-code

BCD	Excess-3
0 - 0000	→ 0011
1 - 0001	→ 0100
2 - 0010	→ 0101
3 - 0011	→ 0110
4 - 0100	→ 0111
5 - 0101	→ 1000
6 - 0110	→ 1001
7 - 0111	→ 1010
8 - 1000	→ 1011
9 - 1001	→ 1100

### Excess-3 addition

In excess-3 addition after adding two excess-3 numbers there are two cases

Case (i): No carry

If there is no carry after excess-3 addition subtract (0011)<sub>3</sub> from the result to get final result

Case (ii): Carry generated:

After excess-3 addition if carry generated add 3 (0011) to both sum & carry.

Ex:

$$\begin{array}{r}
 3 \rightarrow 0110 \\
 (+) 4 \rightarrow (+) 0111 \\
 \hline
 7 \quad 101 \\
 (1010) \quad 0011 \text{ (sub-3)} \\
 \hline
 1010
 \end{array}$$

$$\begin{array}{r}
 2 \rightarrow 0101 \\
 +3 \rightarrow 0110 \\
 \hline
 5 \rightarrow 1011 \\
 1000 \quad 0011 \text{ (Sub 3)} \\
 \hline
 1000
 \end{array}$$

$$\begin{array}{r}
 6 \rightarrow 1001 \\
 7 \rightarrow 1010 \\
 \hline
 13 \rightarrow 0001 \quad 0011 \\
 0011 \quad 0011 \text{ Add 3} \\
 \hline
 0100 \quad 0110 \\
 \hline
 3
 \end{array}$$

$$\begin{array}{r}
 13 \rightarrow 0100 \quad 0110 \\
 12 \rightarrow 0100 \quad 0101 \\
 \hline
 25 \rightarrow 1000 \quad 1011 \\
 \quad \quad - 0011 \text{ Sub(3)} \\
 \hline
 1000 \quad 1000 \\
 \text{Sub 3} \\
 0011 \\
 \hline
 0101 \quad 1000 \\
 \hline
 2 \quad 5
 \end{array}$$

$$\begin{array}{r}
 38 \rightarrow 0110 \quad 1011 \\
 +25 \rightarrow 0101 \quad 1000 \\
 \hline
 63 \rightarrow 1100 \quad 0011 \\
 1001 \quad 0110 \\
 \quad \quad (-) 0011 \quad 0011 \text{ Sub 3} \\
 \hline
 1001 \quad 0110
 \end{array}$$

$$\begin{array}{r}
 45 \rightarrow 0111 \quad 1000 \\
 +36 \rightarrow 0110 \quad 1001 \\
 \hline
 81 \rightarrow 1110 \quad 0001 \\
 \quad \quad \quad \quad 0011 \text{ Add 3} \\
 \hline
 1110 \quad 0100 \\
 \text{Sub 3 } 0011 \\
 \hline
 1011 \quad 0100
 \end{array}$$

$$\begin{array}{r}
 93 \rightarrow 01100 \quad 0110 \\
 48 \rightarrow 01111 \quad 1011 \\
 \hline
 141 \rightarrow 00010100 \quad 0001 \\
 \quad \quad \quad \quad 0011 \quad 0011 \quad 0011 \text{ Add (3)} \\
 \hline
 0100 \quad 0111 \quad 0100 \\
 \hline
 0100 \quad 0111 \quad 0100
 \end{array}$$





$$\begin{array}{r}
 *5 \\
 -4 \xrightarrow{9's\ complement} \\
 \hline
 1
 \end{array}
 \quad
 \begin{array}{r}
 1000 \\
 (+) 1000 \\
 \hline
 10000
 \end{array}
 \quad
 \begin{array}{r}
 9 \\
 4 \\
 \hline
 5
 \end{array}$$

0100

end around carry.

$$\begin{array}{r}
 0001 \\
 0011 \text{ Add 3(0011)} \\
 \hline
 0100
 \end{array}$$

$$\begin{array}{r}
 4 \\
 -5 \xrightarrow{9's\ complement} \\
 \hline
 -1
 \end{array}
 \quad
 \begin{array}{r}
 0111 \\
 0111 \\
 \hline
 1111
 \end{array}
 \quad
 \begin{array}{r}
 9 \\
 5 \\
 \hline
 4
 \end{array}$$

-0100

1 to 0, 0 to 1

Sub 3(0011)

$$\begin{array}{r}
 0011 \\
 1011 \\
 \hline
 0100
 \end{array}$$

9's complement (0100)

$$\begin{array}{r}
 * 26 \rightarrow 0101 \quad 1001 \rightarrow 0101 \quad 1001 \\
 - 14 \xrightarrow{9's} 0100 \quad 0111 \xrightarrow{1's\ comp} 1011 \quad 1000 \\
 \hline
 12
 \end{array}$$

0100 0101

$$\begin{array}{r}
 0001 \quad 0001 \quad 0001 \\
 (+) 1111 \\
 \hline
 0001 \quad 0001 \quad 0001
 \end{array}$$

$$\begin{array}{r}
 * 14 \rightarrow 0100 \quad 0111 \\
 - 26 \xrightarrow{9's} 1010 \quad 0110 \\
 \hline
 -12
 \end{array}$$

$$\begin{array}{r}
 1010 \quad 0110 \\
 (+) 1111 \\
 \hline
 1110 \quad 1101 \\
 0011 \text{ (sub 3)} \\
 \hline
 1110 \quad 1010 \\
 0011 \\
 \hline
 1011 \quad 1010
 \end{array}$$

$$\begin{array}{r}
 0001 \quad 0010 \\
 (+) 0011 \text{ (0011) Add 3} \\
 \hline
 0001 \quad 0101 \\
 0011 \text{ Add 3} \\
 \hline
 0100 \quad 0101 \\
 1 \quad 2
 \end{array}$$

$$\begin{array}{r}
 99 \\
 26 \\
 \hline
 73
 \end{array}$$

9's of result (-0100 0101)

$$\begin{array}{r}
 0100 \quad 0101 \\
 1 \quad 2
 \end{array}$$

\* 2 5 5 → 0101 1000 1000 → 0101 1000 1000  
 (-) 1 6 3 → 0100 1001 0110 → 1011 0110 1001  
0 9 2

0011 1100 0101

(+)  
 1 0 0 0 0 1 1 1 1 0 0 0 1  
 0 0 0 0 1 1 1 1 0 0 1 0  
 0 0 1 1 0 0 1 1 0 0 1 1  
 0 0 1 1 0 0 1 1 0 0 1 1  
 0 0 0 0 1 1 1 1 0 1 0 1  
 0 0 1 1  
0 0 0 0 1 1 1 1 0 1 0 1  
 (Add3) 0 0 1 1 0 0 1 1  
 (-) 0 0 1 1  
0 0 1 1 1 0 0 0 0 1 0 1  
0 9 2

→ 1 6 3 → 0100 1001 0110  
 (-) 2 5 5<sup>15</sup> → 1010 0111 0111  
- 0 9 2

(+)  
 1 1 1 1 0 0 0 0 1 1 0 1  
 0 0 1 1 + 0 0 1 1 (-) 0 0 1 1  
1 1 0 0 0 0 0 0 0 0 0 0  
 0 0 1 1 1 0 1 0

1's complement  
 (-0 0 1 1 1 1 0 0 0 1 0 1)

\* 2 17 33 88 15 → 0110 1011 1000 999  
 - 1 9 6 → 1011 0011 0110 196  
1 8 9 803

0100 1011 1100

(+)  
 0 0 0 1 1 1 0 0 1 1 1 1 0 1  
 0 0 1 1 0 0 1 1 (-) 0 0 1 1 Add3  
0 1 0 0 1 0 1 1 1 1 0 0  
1 8 9

196 → 0100 1100 1001  
 -385 1001 0100 0111  
 -189 1110 0001 0000  
 0100 1011 (-) 0011 (+) 0011 (+) 0011  
 1100 1011 0100 0011

q's of result: (0100 1011 1100)  
 1 8 9

*[Faded handwritten notes and diagrams, including a large diagonal line and various binary strings]*

# Signed Binary numbers:

In signed binary numbers, one sign bit is allocated for representation of +ve numbers and -ve numbers. For +ve numbers sign bit is 0. For -ve numbers the sign bit is 1.

For EX:  
 =  
 0 0001 = +1  
 1 0001 = -1

\* The signed binary numbers can be represented in three ways -

- (1) Sign magnitude representation
- (2) 1's complement "
- (3) 2's Complement "

Note:  
 \*\*\*  
 \*\*\*  
 \*\*\*

In all three representations, +ve numbers are having unique representation, where as -ve numbers having different representations

EX:	<u>+5</u>	<u>-5</u>
sign magnitude	0 0000101	1 0000101
1's complement	0 0000101	1 1111010
2's complement	0 0000101	1 1111011

EX: using 2's complement

- +7 = 00000111
- +5 = 00000101
- 5 = 11111011
- 7 = 11111001
- +2 = 00000010
- 2 = 11111110
- +12 = 00001100
- 12 = 11110100

+7	→	00000111
+5	→	00000101
		+12
		00001100
		111
		00001100

+7	→	00000111
-5	→	11111011
+2	→	00000010
		discarded 1
		00000010
-7	→	11111001
+5	→	00000101
-2	→	11111110

$$\begin{array}{r}
 +7 \rightarrow 11111001 \\
 -5 \rightarrow 11111011 \\
 \hline
 -12 \xrightarrow{(+)} 11111011 \\
 \text{discarded } \otimes 11110100
 \end{array}$$

some above question using 1's complement

\* Using 1's Complement

$$\begin{array}{l}
 +7 \rightarrow 00000111 \\
 +5 \rightarrow 00000101 \\
 -7 \rightarrow 11111000 \\
 -5 \rightarrow 11111010 \\
 +2 \rightarrow 00000010 \\
 -2 \Rightarrow 1111101 \\
 +12 \rightarrow 00001100 \\
 -12 \rightarrow 11110011
 \end{array}$$

$$\begin{array}{r}
 +7 \rightarrow 00000111 \\
 +5 \rightarrow 00000101 \\
 \hline
 +12 \xrightarrow{(+)} 111 \\
 00001100
 \end{array}$$

$$\begin{array}{r}
 +7 \rightarrow 00000111 \\
 -5 \rightarrow 11111010 \\
 \hline
 +2 \xrightarrow{(+)} 10000001 \\
 \hline
 00000010
 \end{array}$$

$$\begin{array}{r}
 -7 \rightarrow 11111000 \\
 +5 \rightarrow 00000101 \\
 \hline
 -2 \xrightarrow{(+)} 1111101
 \end{array}$$

$$\begin{array}{r}
 -7 \rightarrow 11111000 \\
 -5 \rightarrow 11111010 \\
 \hline
 -12 \xrightarrow{(+)} 111110010 \\
 \hline
 11110011
 \end{array}$$

L

## ⇒ ERROR DETECTING CODES

(39)

Parity bit is an extra bit which is added to the original bit. Parity is of two types

1. Even Parity.
2. Odd Parity.

Even Parity: For even Parity the no. of 1's in the information should be even including parity bit.

Odd Parity: For odd Parity the no. of 1's in information is odd including Parity bit.

Ex: Generate Even & odd Parity for given message.

Ans → 0 0 1 0 1 1 0

0 0 1 0 1 1 0 ① → even parity

0 0 1 0 1 1 0 ② → odd parity

Block Parity: In block Parity the parity is taken for both rows & columns for group of messages.

Ex:

Even parity

0	0	1	0	1	0		0	
0	0	1	0	1	0		0	
0	0	1	0	1	1		1	
1	0	1	0	1	0		1	
<hr/>								
1	0	0	0	0	1			

# Error detection & correction codes (40)

In the error detection & correction codes Hamming code is the best example. By using Hamming code, error can be detected and it can be corrected.

## Hamming code generation:

To generate Hamming code for  $m$  no. of message bits the following formula is used

$$2^p \geq m + p + 1$$

where  $p$  = no. of parity bits

Generate Hamming code for 1011.

Ans

$$\begin{array}{cccc} 1 & 0 & 1 & 1 \\ m_1 & m_2 & m_3 & m_4 \\ m = 4 \end{array}$$

$$2^p \geq m + p + 1$$

$$2^p \geq 4 + p + 1 \quad [\because m = 4]$$

if  $p = 0$

$$2^0 \geq 4 + 0 + 1$$

$$1 \geq 5$$

if  $p = 1$

$$2^1 \geq 4 + 1 + 1$$

$$2 \geq 6$$

if  $p = 2$

$$2^2 \geq 4 + 2 + 1$$

$$4 \geq 7$$

if  $p = 3$

$$2^3 \geq 4 + 3 + 1$$

$$8 \geq 8$$

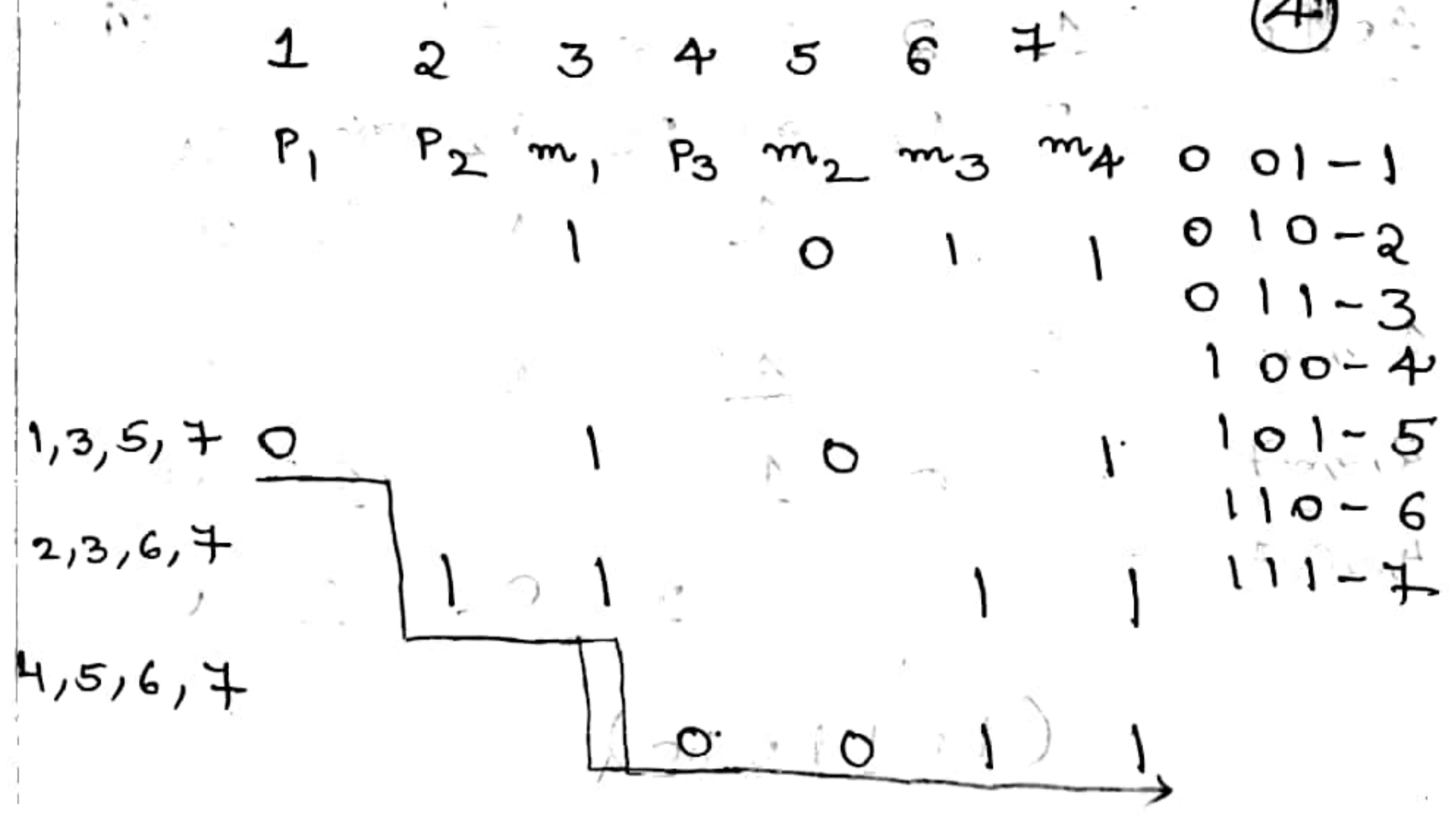
$$m = 4$$

$$p = 3$$

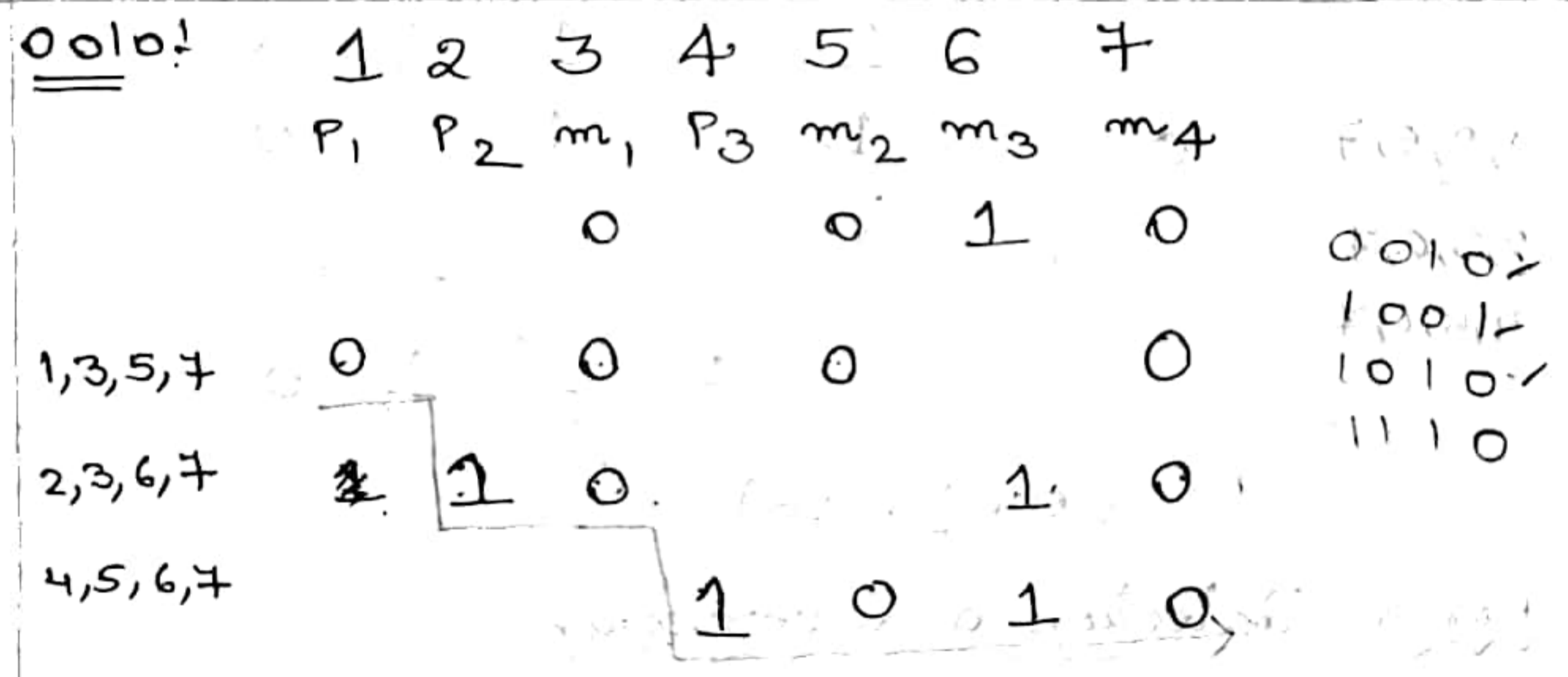
$$\text{Total} = \underline{\underline{7}}$$



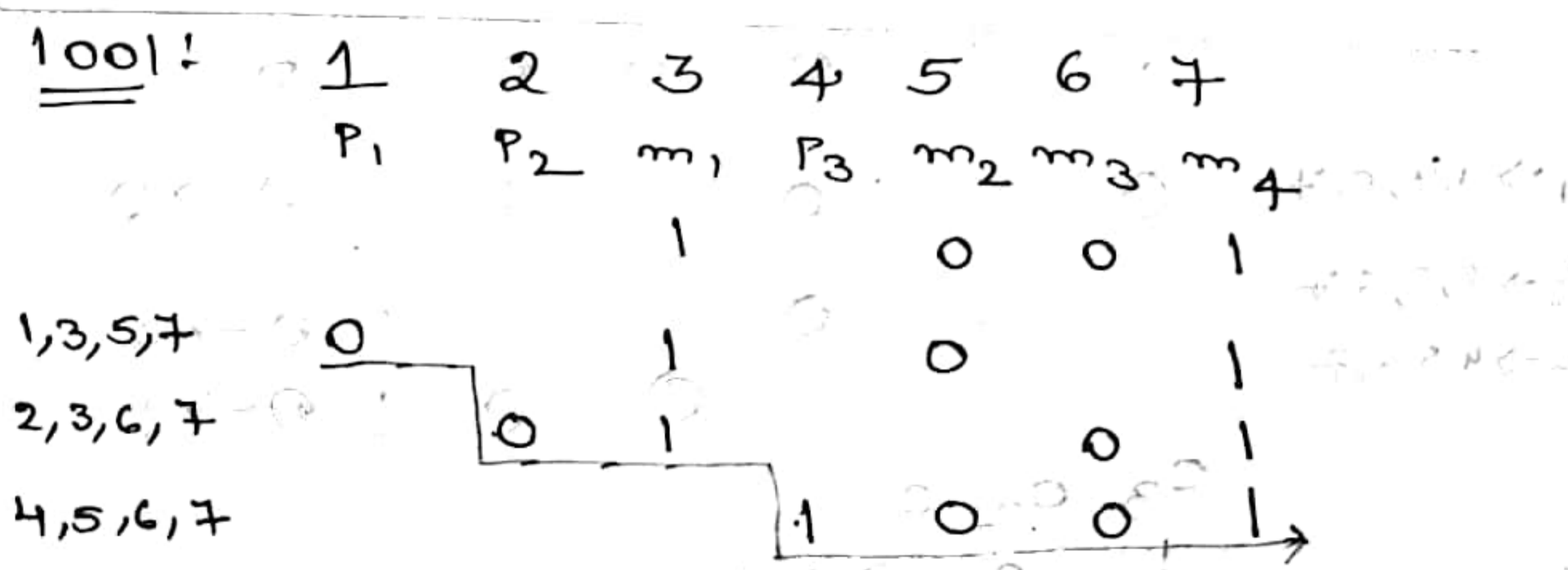
(4)



(011011)



(0101010)



(0011001)

1010!

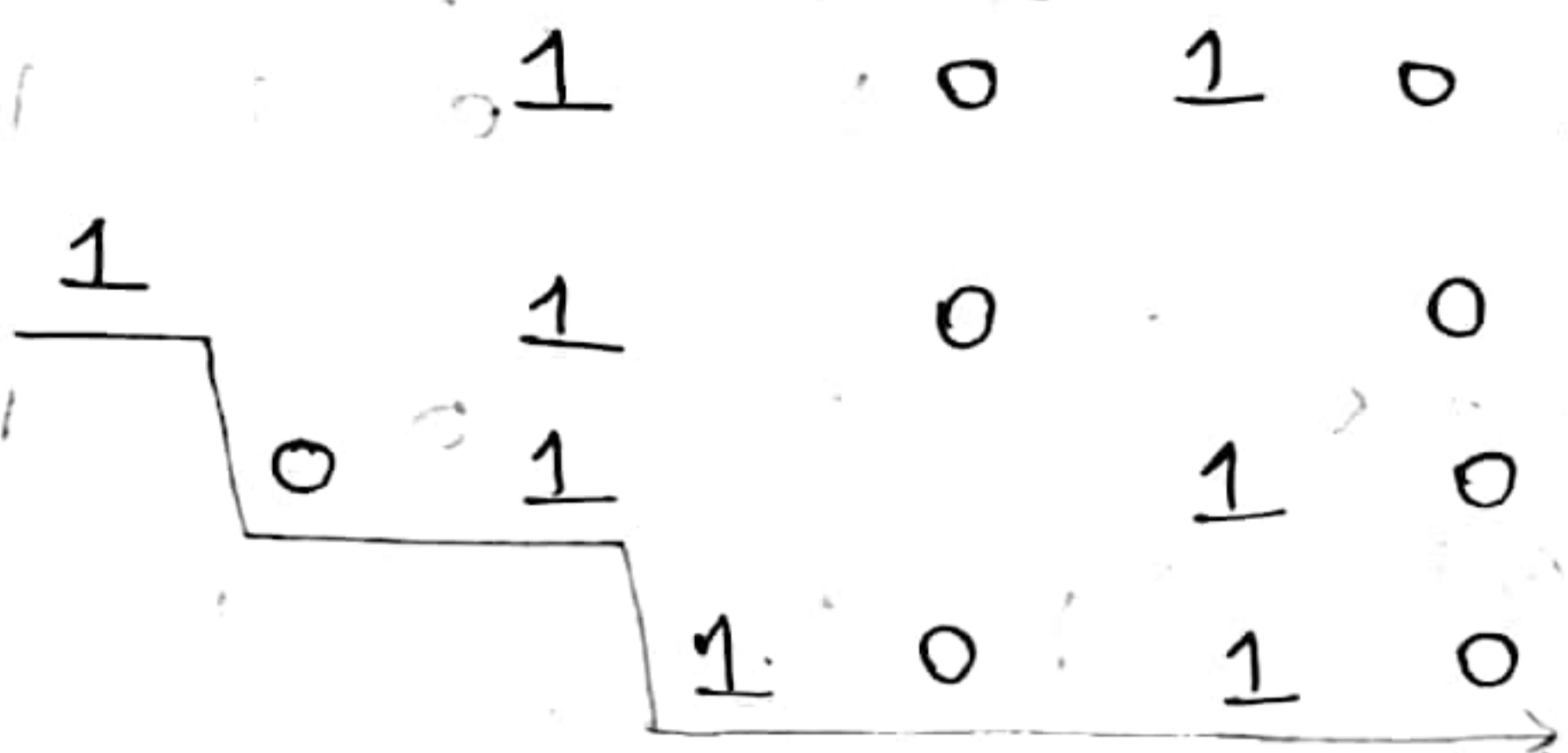
1 2 3 4 5 6 7  
P<sub>1</sub> P<sub>2</sub> m<sub>1</sub> P<sub>3</sub> m<sub>2</sub> m<sub>3</sub> m<sub>4</sub>

(4,2)

1,3,5,7

2,3,6,7

4,5,6,7



(1011010)

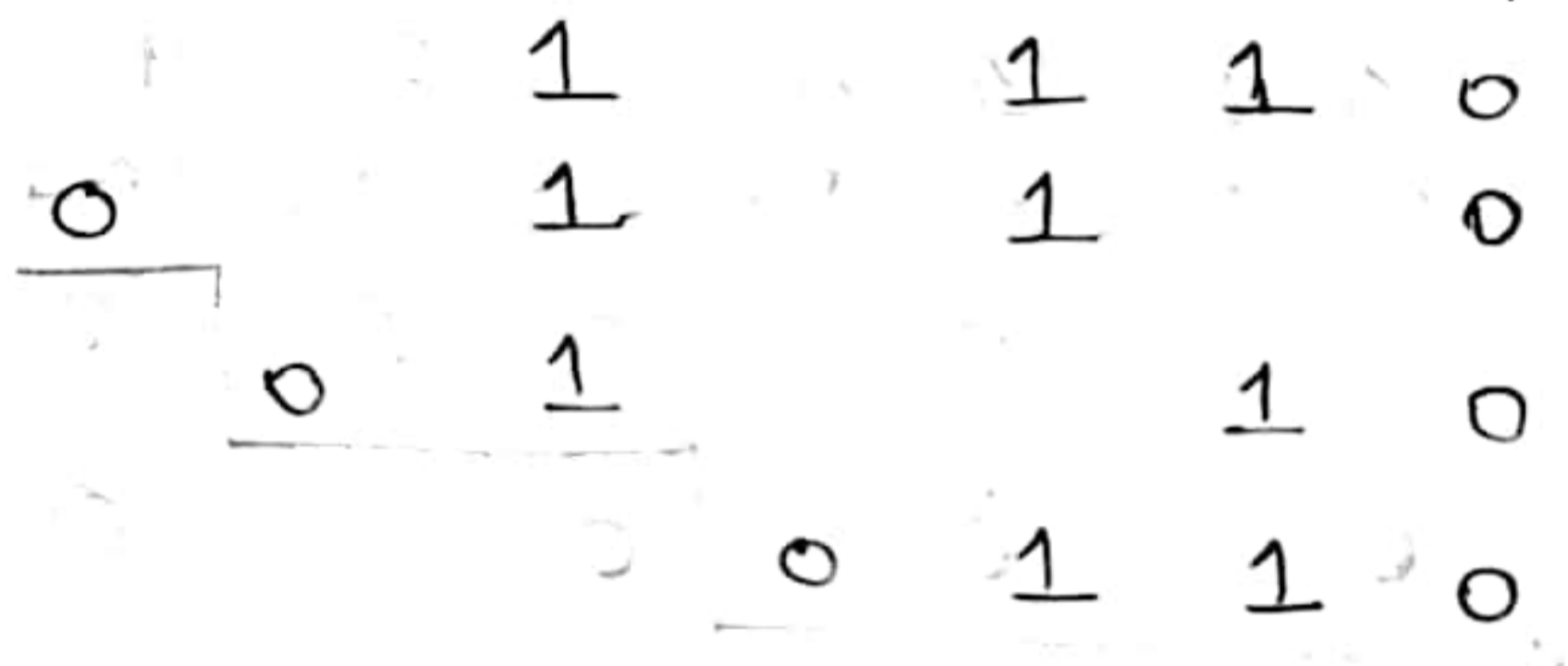
1110!

1 2 3 4 5 6 7  
P<sub>1</sub> P<sub>2</sub> m<sub>1</sub> P<sub>3</sub> m<sub>2</sub> m<sub>3</sub> m<sub>4</sub>

1,3,5,7

2,3,6,7

4,5,6,7



(0010110)

Error detection and correction:

→ original: 0101010  $\xrightarrow{err}$  0100010

	1	2	3	4	5	6	7
	0	1	0	0	0	1	0
C <sub>1</sub> → 1,3,5,7	0		0		0		0 → 0
C <sub>2</sub> → 2,3,6,7		1	0			1	0 → 0
C <sub>3</sub> → 4,5,6,7				0	0	1	0 → 1

C<sub>3</sub> C<sub>2</sub> C<sub>1</sub>  
1 0 0 ⇒ 4

∴ There is an error in 4<sup>th</sup> position.  
the correct information is 0101010

⇒ 0101010 <sup>error</sup> ⇒ 0101000

	1	2	3	4	5	6	7	
	0	1	0	1	0	0	0	
$C_1 \rightarrow 1, 3, 5, 7 \rightarrow$	0		0		0		0	$\rightarrow 0$
$C_2 \rightarrow 2, 3, 6, 7 \rightarrow$		1	0			0	0	$\rightarrow 1$
$C_3 \rightarrow 4, 5, 6, 7 \rightarrow$				1	0	0	0	$\rightarrow 1$

$C_3 \ C_2 \ C_1$   
1 1 0  $\rightarrow 6$

∴ There is an error in 6<sup>th</sup> position.  
The correct information is 0101010

⇒ 0101010 <sup>error</sup> ⇒ 0001010

	1	2	3	4	5	6	7	
	0	0	0	1	0	1	0	
$C_1 \rightarrow 1, 3, 5, 7 \rightarrow$	0		0		0		0	$\rightarrow 0$
$C_2 \rightarrow 2, 3, 6, 7 \rightarrow$		0	0			1	0	$\rightarrow 1$
$C_3 \rightarrow 4, 5, 6, 7 \rightarrow$				1	0	1	0	$\rightarrow 0$

$C_3 \ C_2 \ C_1$   
0 1 0  $\rightarrow 2$

∴ There is an error in 2<sup>nd</sup> position, The correct information is 0101010.

⇒ 0101010

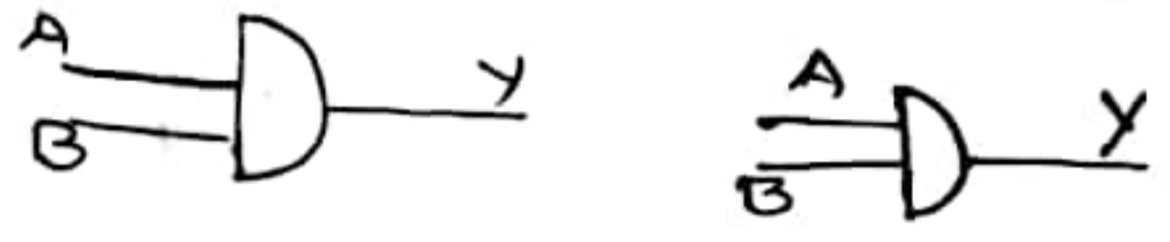
	1	2	3	4	5	6	7	
	0	1	0	1	0	1	0	
$C_1 \rightarrow 1, 3, 5, 7$	0		0		0		0	$\rightarrow 0$
$C_2 \rightarrow 2, 3, 6, 7$		1	0			1	0	$\rightarrow 0$
$C_3 \rightarrow 4, 5, 6, 7$				1	0	1	0	$\rightarrow 0$

$C_3 \ C_2 \ C_1$   
0 0 0  
No error.

# Logic Gates:

- (1) Basic gates  $\rightarrow$  AND, OR, NOT
- (2) Universal gates  $\rightarrow$  NAND, NOR
- (3) Special gates  $\rightarrow$  EX-OR, EX-NOR

## AND Gate



A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

$$Y = A \cdot B$$
$$= AB$$

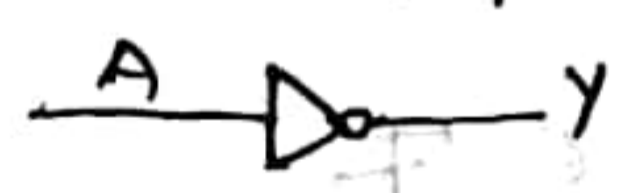
## OR Gate



A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

$$Y = A + B$$

## NOT Gate



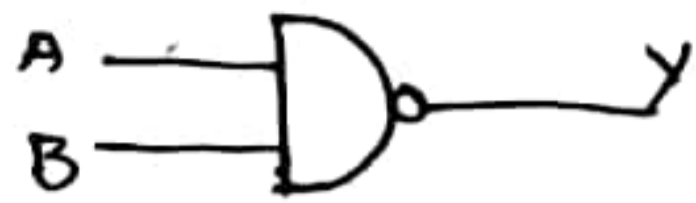
A	Y
0	1
1	0

$$Y = \bar{A} = A'$$

## NAND Gate :

(45)

NAND Gate is • AND gate followed by NOT Gate



$$Y = \overline{A \cdot B}$$
$$= \overline{AB}$$

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

Any boolean function can be  
fully implemented with this  
two gates (NAND, NOR)

## NOR Gate :

NOR Gate is OR Gate followed by NOT Gate.



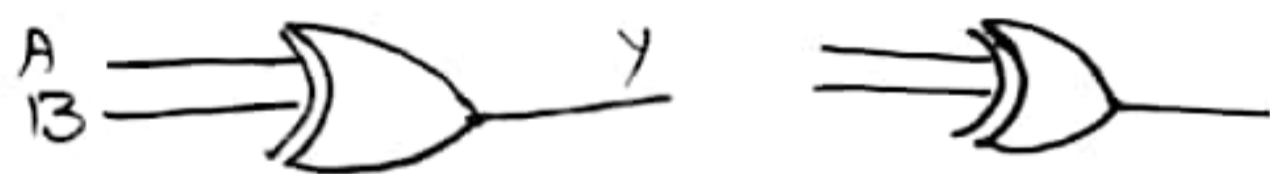
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

$$Y = \overline{(A+B)}$$



## Special gates:

→ EX-OR: (inequality gate)



A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

$$Y = A \oplus B$$
$$= A\bar{B} + \bar{A}B$$

Important:

Property:

$$A \oplus 0 = A$$
$$A \oplus 1 = \bar{A}$$

# EX-NOR (equality gate)

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A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

$$\begin{aligned}
 Y &= A \odot B \\
 &= AB + \overline{AB} \\
 &= AB + \overline{A}\overline{B}
 \end{aligned}$$

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1